Accelerating Predicate Abstraction for Probabilistic Automata

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RWTH Aachen University

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Motivation

Why Model Checking?

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- testing cannot prove absence of bugs

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- formal proof

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Verification of probabilistic models, e.g. network protocols

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Why Probabilistic Model Checking?

Verification of probabilistic models, e.g. network protocols

Properties

- eventually a collision free transmission occurs
- no collision ever occurs
- probability for a collision is below 5%

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State Space Explosion

Even “simple” system descriptions yield huge state spaces

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$\Rightarrow$ construction & analysis often infeasible (memory & time constraints)

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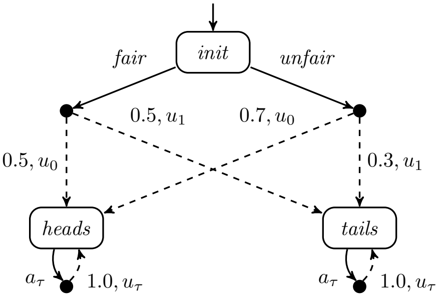
- use space-efficient, symbolic data structures (BDD)

Probabilistic Automaton (Example)



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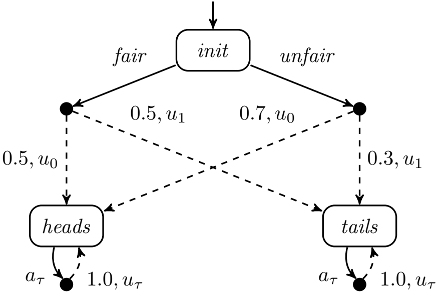

Probabilistic Reachability

- reachability depends on strategy σ of resolving non-determinism

$$Pr^{\sigma}_{\mathcal{A}}(\Diamond G)$$

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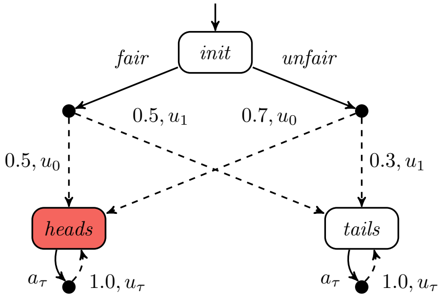

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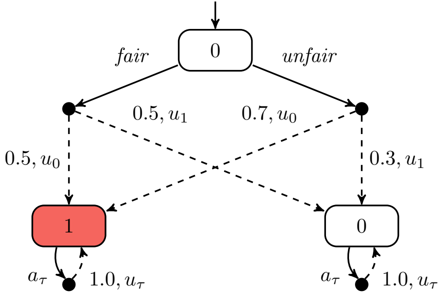

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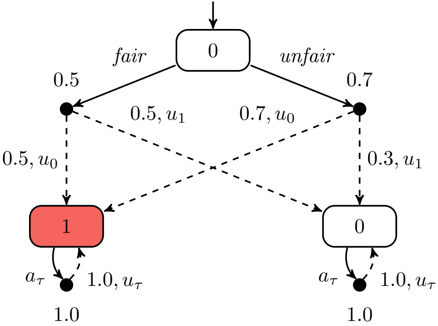

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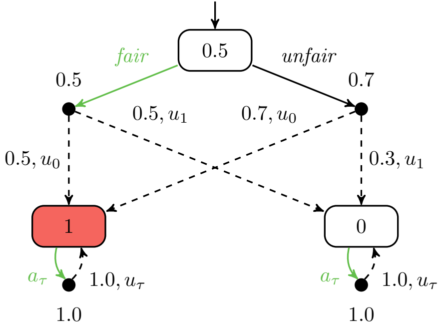

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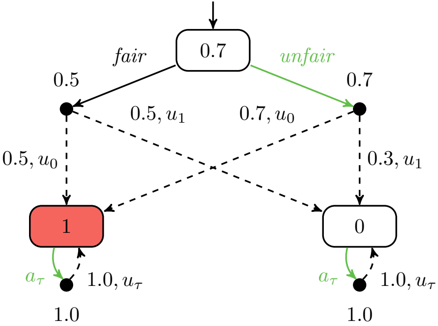

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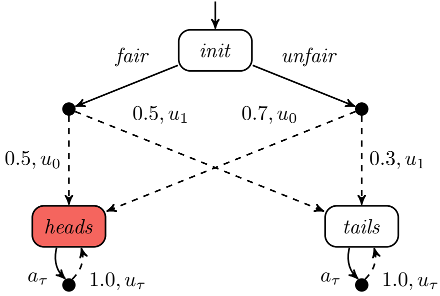

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Probabilistic Program (Example)

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module simple
    // 0=init, 1=running,
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    phase : [0..3];
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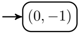

Legend: $(phase, run)$

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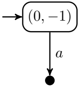

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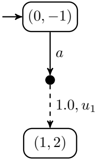

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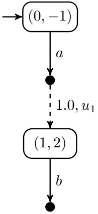

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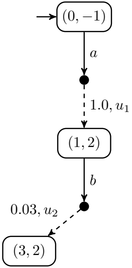

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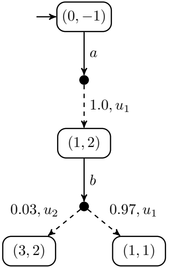

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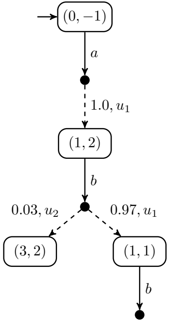

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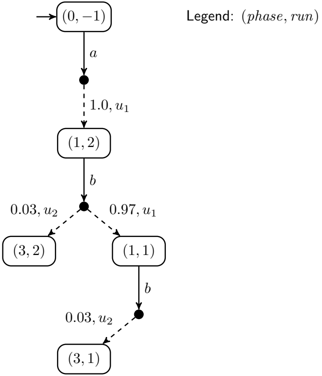

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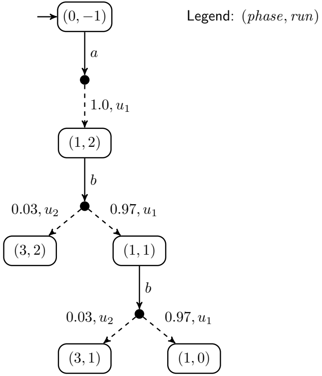

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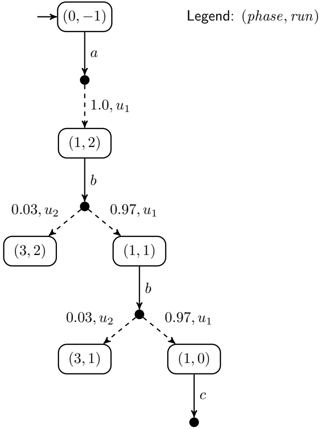

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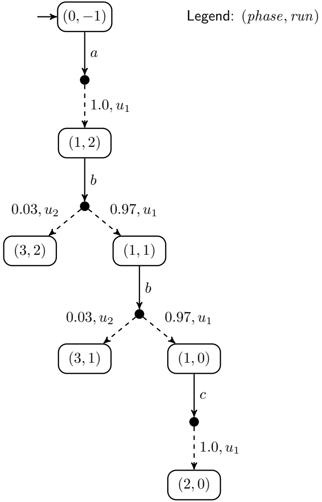

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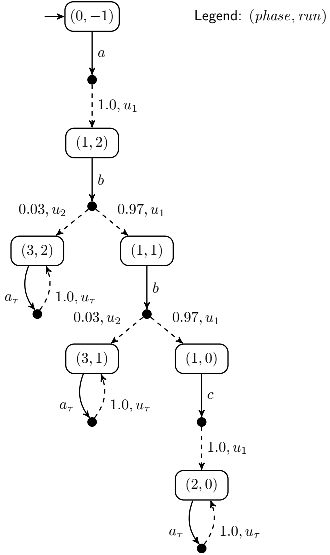
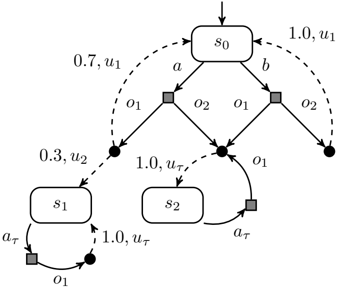


Stochastic Game (Example)



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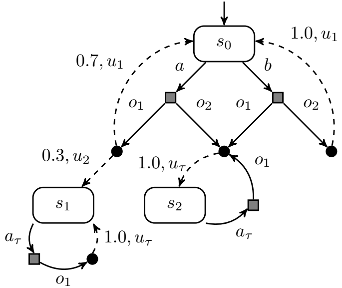

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Reachability probability $Pr_{\mathcal{G}}^{\sigma_1,\sigma_2}(\Diamond G)$ depends on strategy-pair (σ_1, σ_2)

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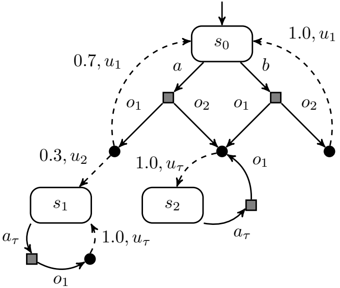

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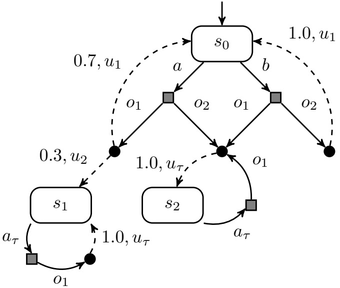

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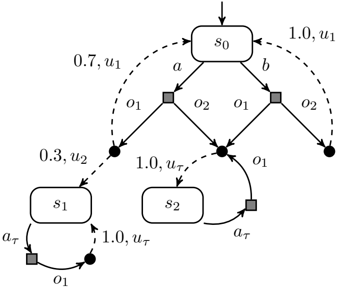

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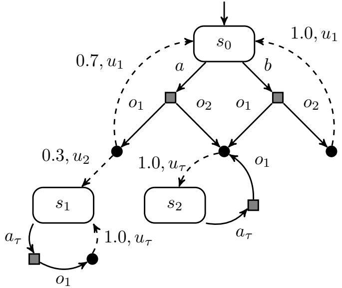

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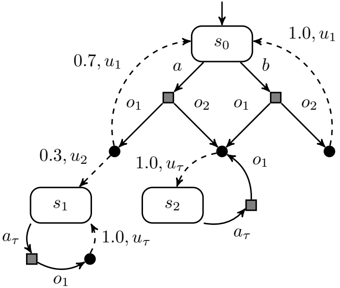

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Multi-Terminal Binary Decision Diagram

DAG $\mathfrak{D}$ representing a function $f_{\mathfrak{D}} : \mathbb{B}^n \to \mathbb{R}$ with finite range

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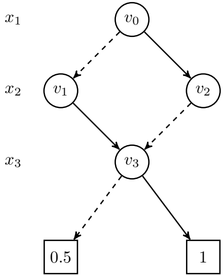

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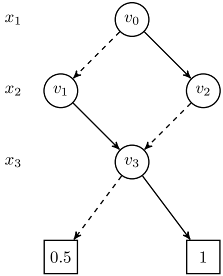

x_1	x_2	x_3	$f_{\mathfrak{D}}$
0	0	0	0
0	0	1	0
0	1	0	0.5
0	1	1	1
1	0	0	0.5
1	0	1	1
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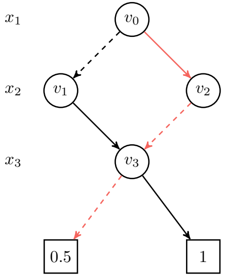

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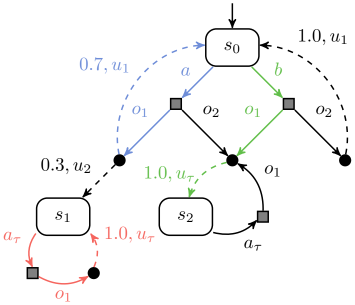
Stochastic Game as MTBDD (Example)



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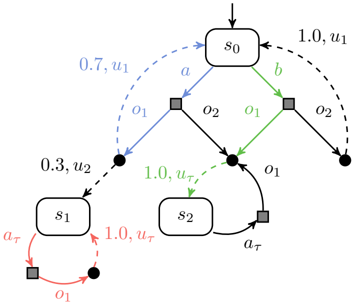
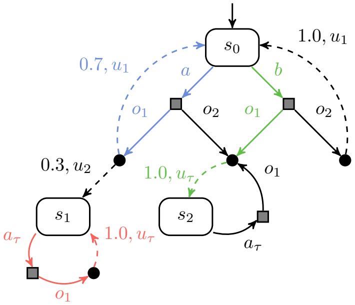


Encoding Excerpt

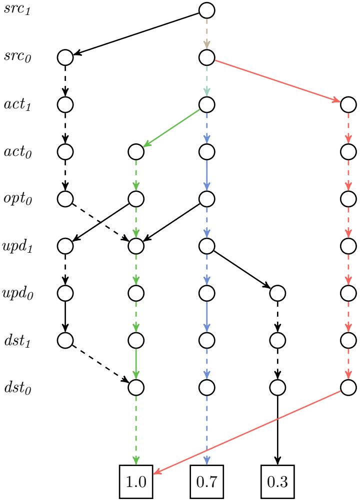
$$\delta(s_0, b, o_1, u_\tau, s_2) = 1.0$$

Stochastic Game as MTBDD (Example)



Encoding Excerpt

$$\delta(s_0, b, o_1, u_\tau, s_2) = 1.0$$



Menu-game: Concept

Partition Abstraction

Partition PA's state space S into blocks Q:

$$S = \biguplus_{B \in Q} B$$

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Non-determinism of Model & Abstraction

- merge non-determinism
- distinguish non-determinism

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Over-approximation [Wachter, 2011]

$$Pr_{\mathcal{G}}^{-,-}\left(\Diamond G^{\#}\right) \;\leq\; Pr_{\mathcal{A}}^{-}(\Diamond G) \;\leq\; Pr_{\mathcal{G}}^{-,+}\left(\Diamond G^{\#}\right)$$

Menu-game: Concept

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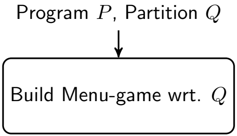
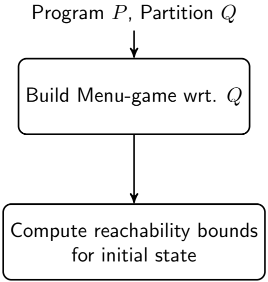
$$\begin{array}{ccccc} Pr_{\mathcal{G}}^{-,-}(\Diamond G^{\#}) & \leq & Pr_{\mathcal{A}}^{-}(\Diamond G) & \leq & Pr_{\mathcal{G}}^{-,+}(\Diamond G^{\#}) \\ Pr_{\mathcal{G}}^{+,-}(\Diamond G^{\#}) & \leq & Pr_{\mathcal{A}}^{+}(\Diamond G) & \leq & Pr_{\mathcal{G}}^{+,+}(\Diamond G^{\#}) \end{array}$$

Backward Refinement Scheme



Backward Refinement Scheme



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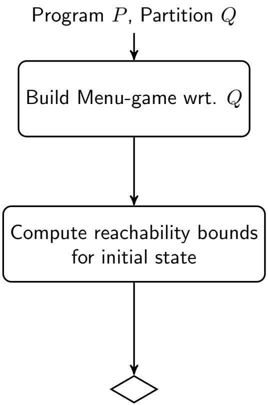

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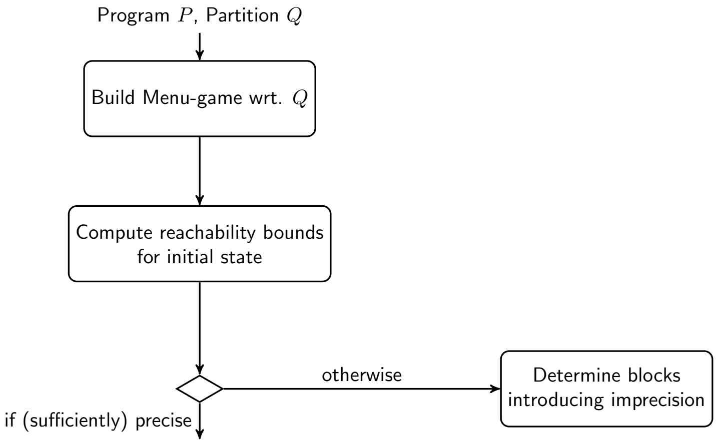

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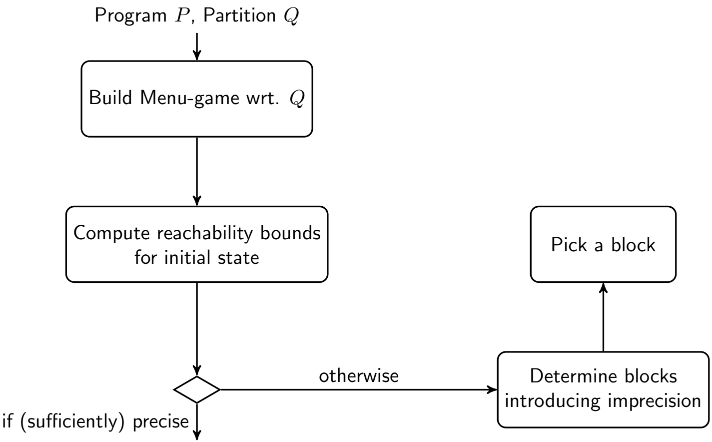

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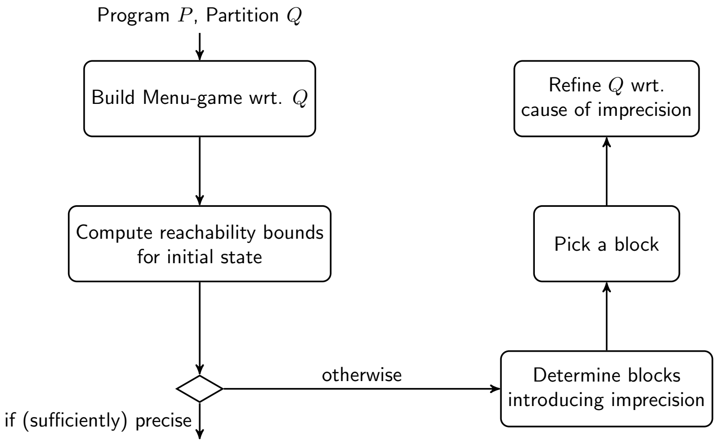
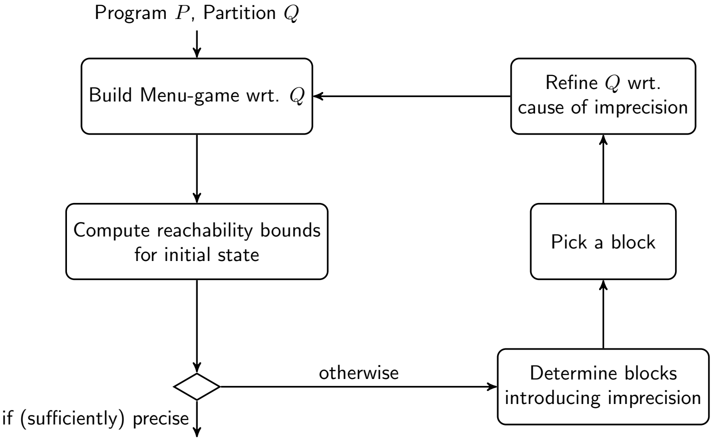
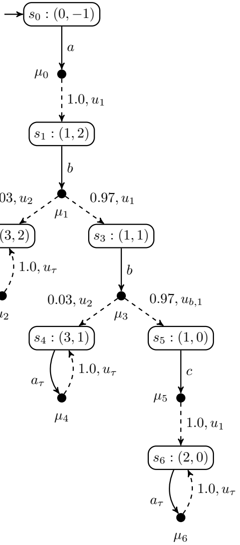


Backward Refinement Scheme



Menu-game: Construction from PA (Example)



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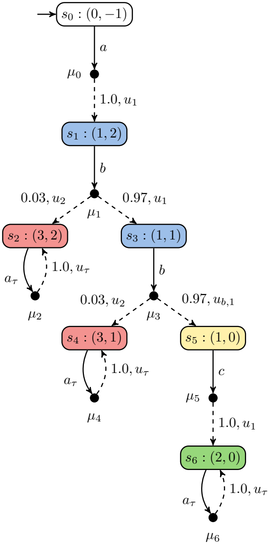

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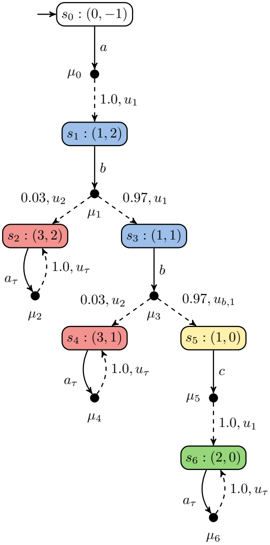

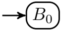

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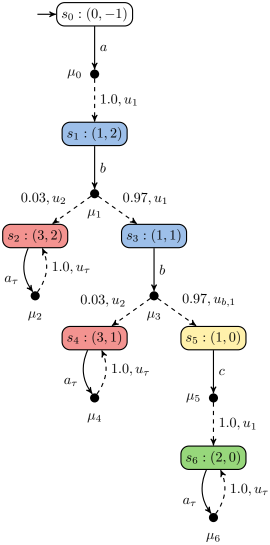

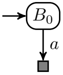

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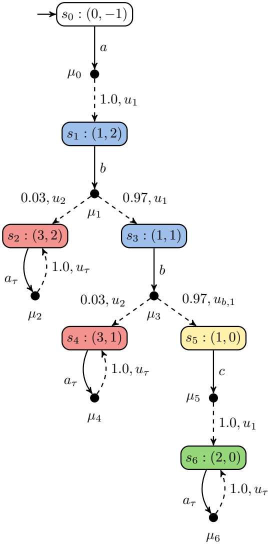

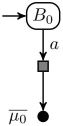

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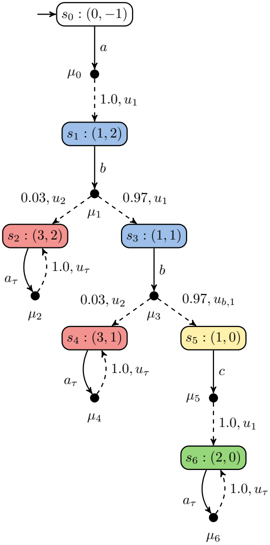

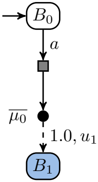

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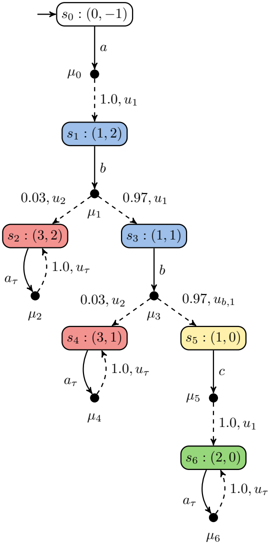

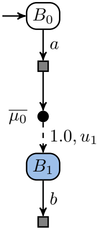

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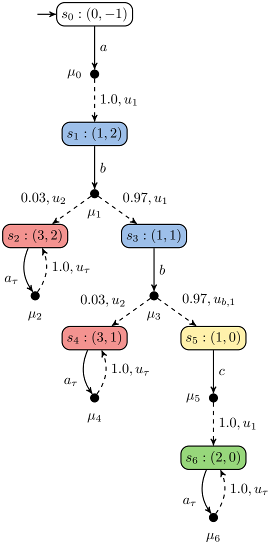

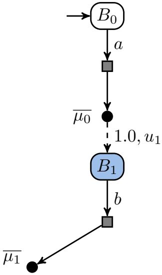

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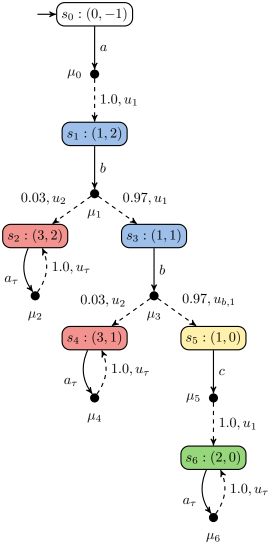

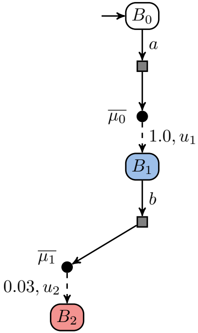

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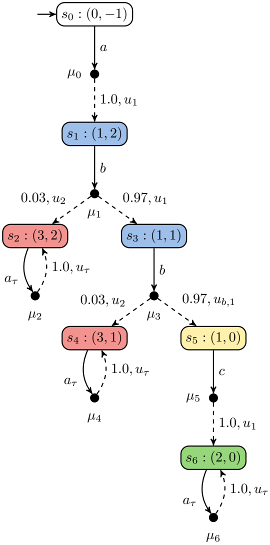

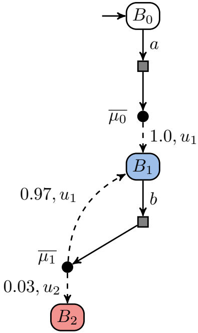

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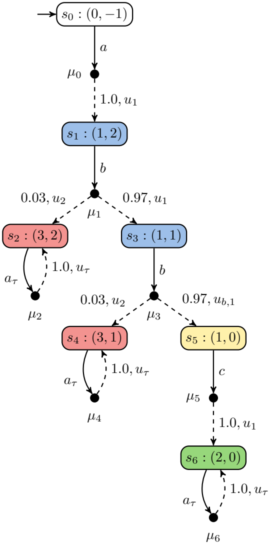

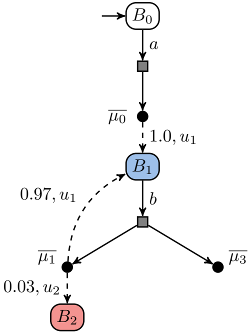

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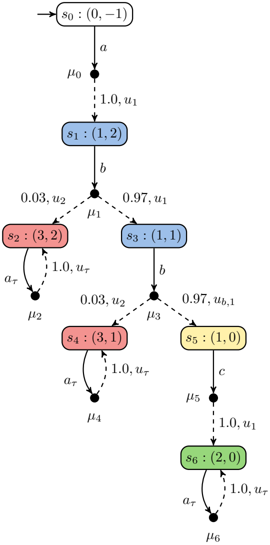

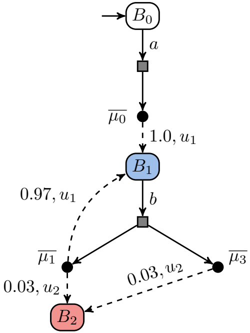

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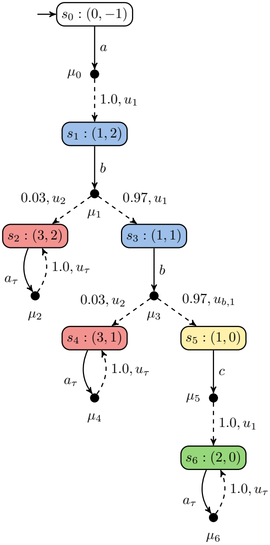

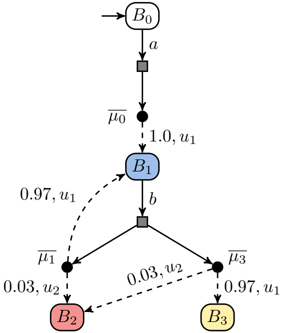

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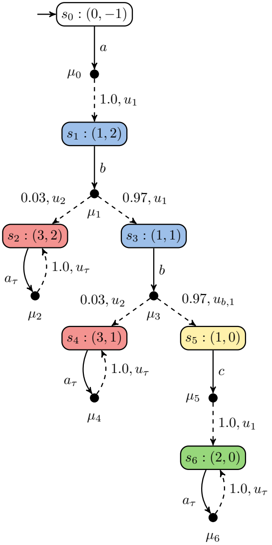

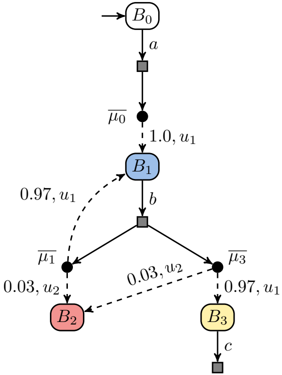

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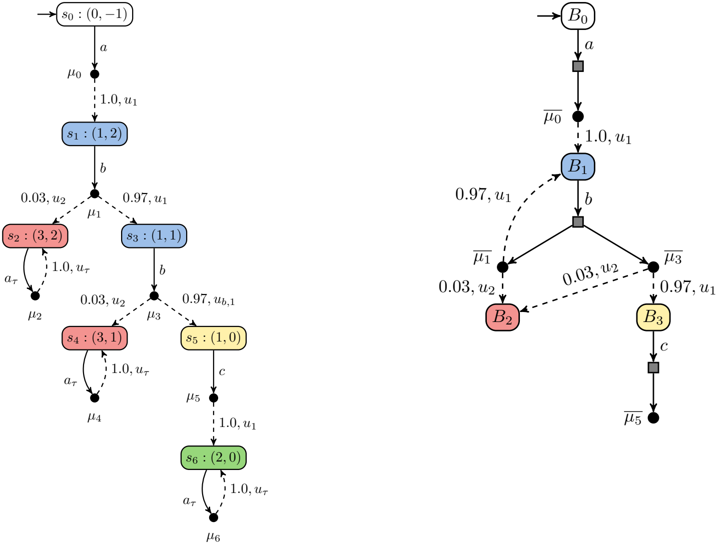

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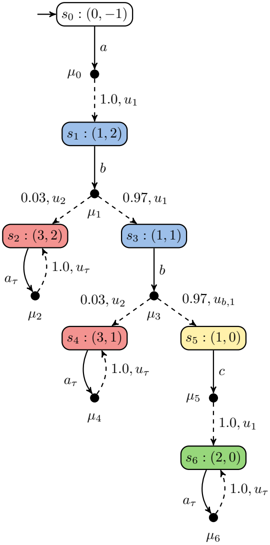

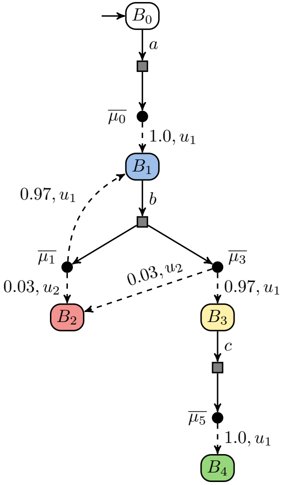

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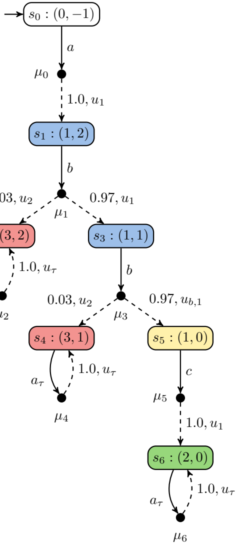

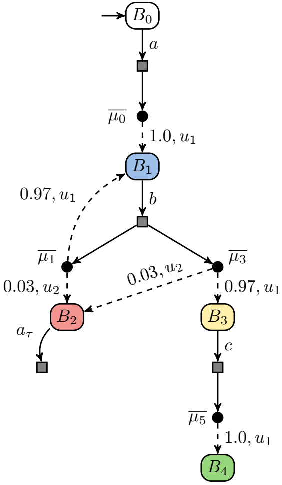

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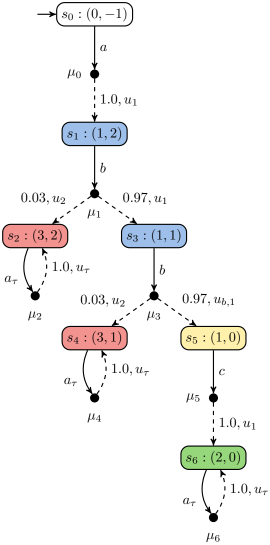

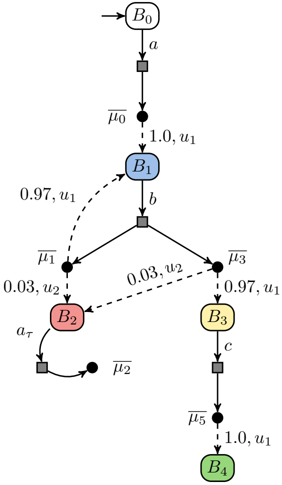

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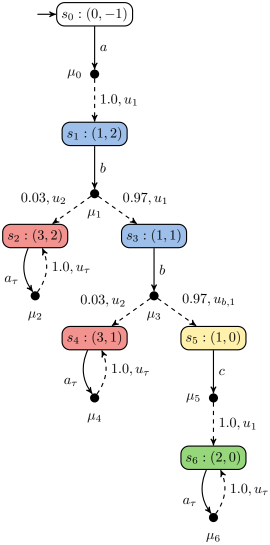

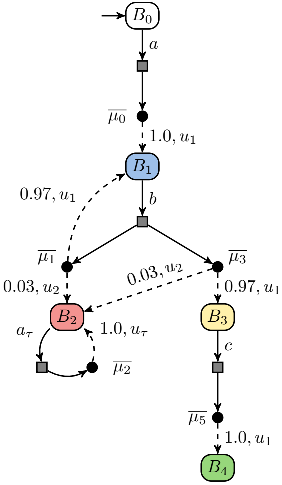

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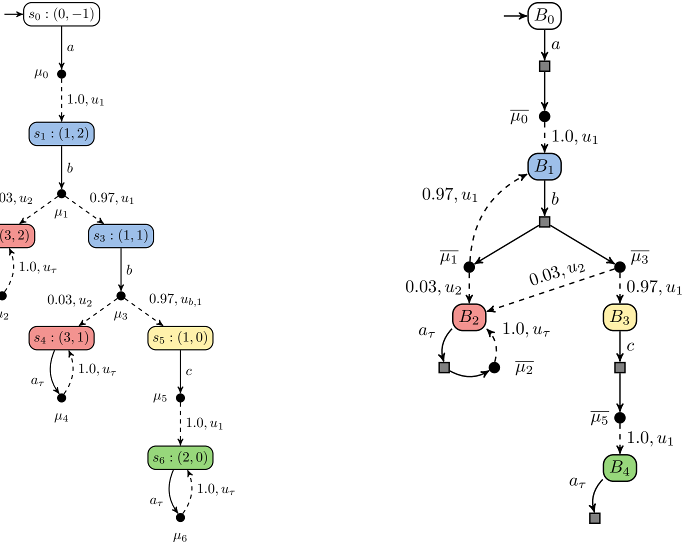

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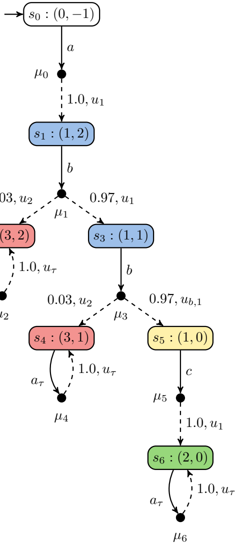

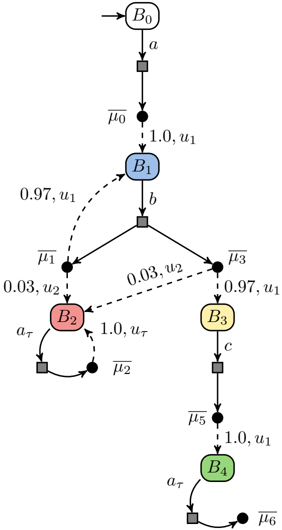

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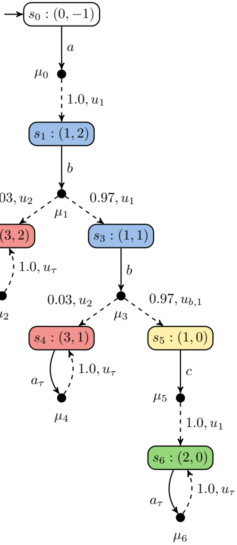
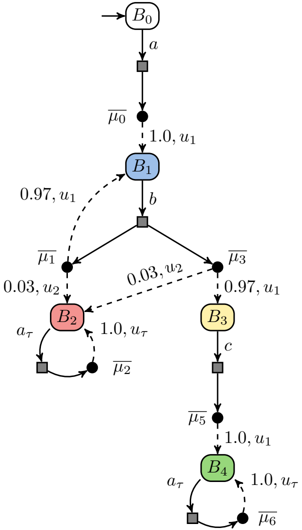
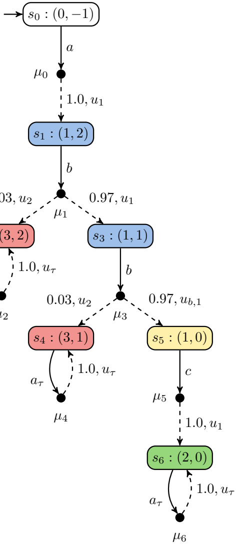




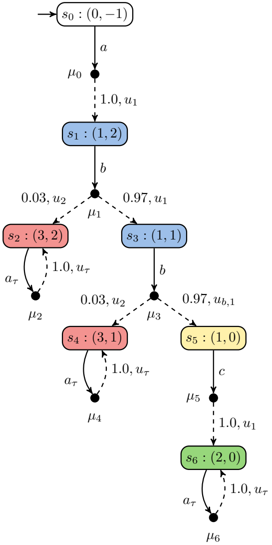
Menu-game: Predicate Abstraction



Predicate

Boolean expression over a program's variables

Menu-game: Predicate Abstraction



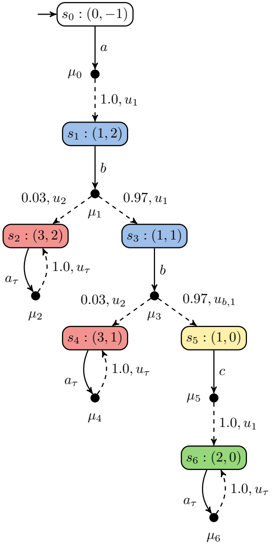
Predicate

Boolean expression over a program's variables

Predicates induce partitioning

$$\mathcal{P} = \{phase = 0, phase = 1, phase = 2, phase = 3, run > 0\}$$

Menu-game: Predicate Abstraction



Predicate

Boolean expression over a program's variables

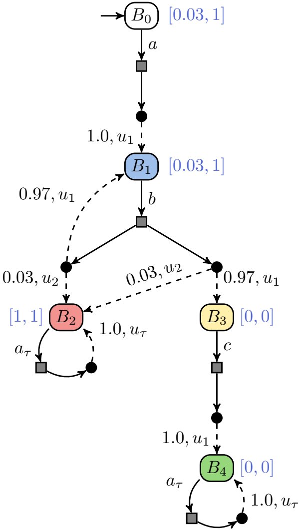
Predicates induce partitioning

$$\mathcal{P} = \{phase = 0, phase = 1, phase = 2, phase = 3, run > 0\}$$

induces the partition:

- $phase = 0$
- $phase = 1, run > 0$
- $phase = 1$
- $phase = 2, run > 0$
- $phase = 3, run > 0$

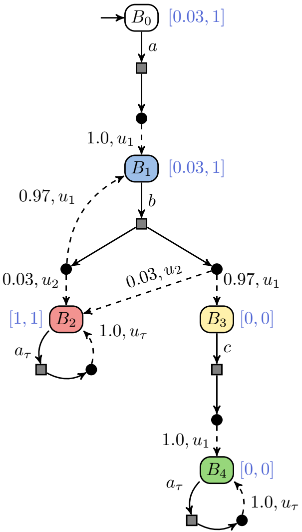
Refinement



Idea

Split *pivot blocks*, which introduce imprecision

Refinement



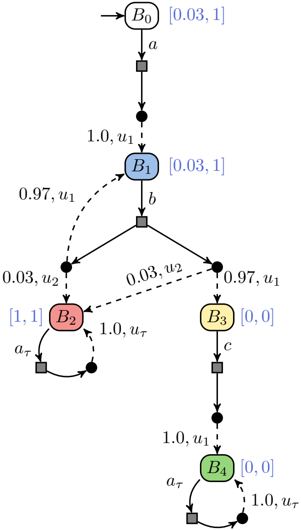
Idea

Split *pivot blocks*, which introduce imprecision

Observations

- deviation alone does not indicate a block being pivot

Refinement



Idea

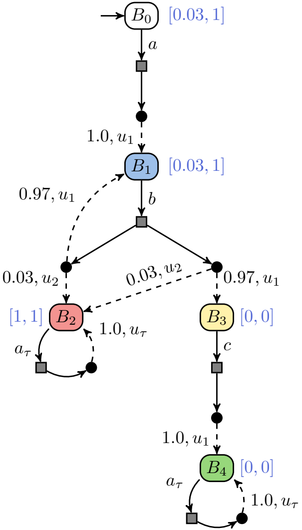
Split *pivot blocks*, which introduce imprecision

Observations

- deviation alone does not indicate a block being pivot

$\Rightarrow$ differing player 2 strategies do

Refinement



Idea

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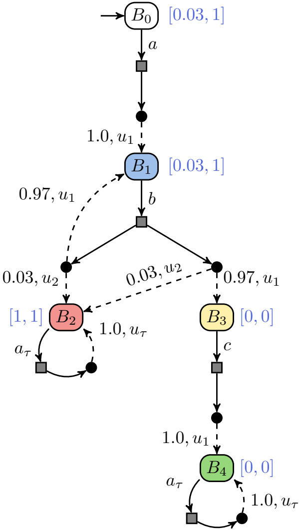
- deviation alone does not indicate a block being pivot

⇒ differing player 2 strategies do

Refinement Predicates

- derived from update leading to different blocks

Refinement



Idea

Split *pivot blocks*, which introduce imprecision

Observations

- deviation alone does not indicate a block being pivot

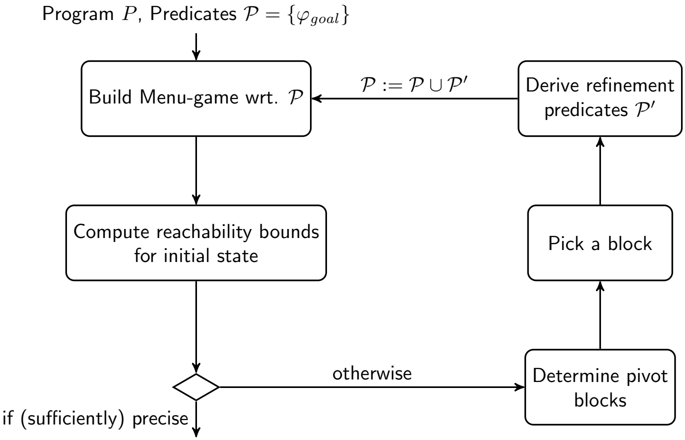
⇒ differing player 2 strategies do

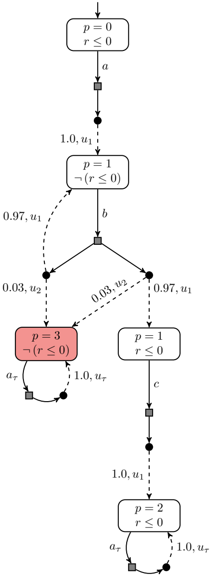
Refinement Predicates

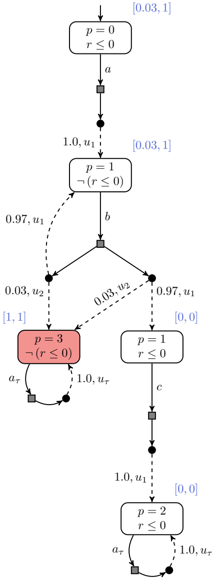
- derived from update leading to different blocks

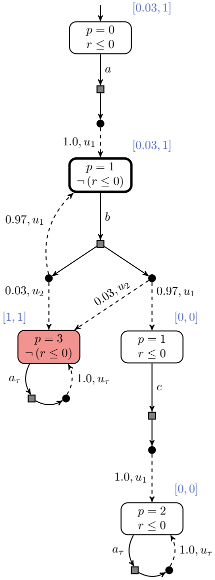
⇒ splitting corresponding behaviours

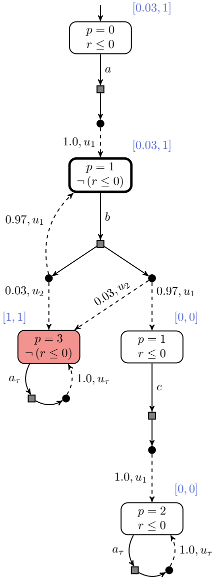
Reminder

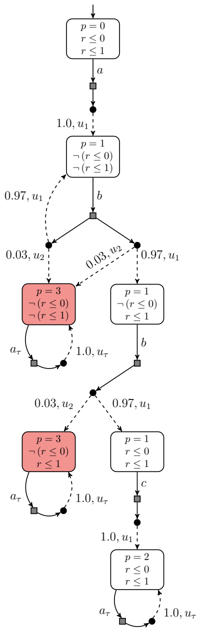

Program P, Predicates $\mathcal{P} = \{\varphi_{goal}\}$
Build Menu-game wrt. $\mathcal{P}$
$\mathcal{P} := \mathcal{P} \cup \mathcal{P}'$
Derive refinement predicates $\mathcal{P}'$
Compute reachability bounds for initial state
Pick a block
otherwise
Determine pivot blocks
if (sufficiently) precise


$p = 0$
$r \leq 0$
a
$1.0, u_1$
$p = 1$
$\neg(r \leq 0)$
$0.97, u_1$
b
$0.03, u_2$
$0.03, u_2$
$0.97, u_1$
$p = 3$
$\neg(r \leq 0)$
a_τ
$1.0, u_\tau$
$p = 1$
$r \leq 0$
c
$1.0, u_1$
$p = 2$
$r \leq 0$
a_τ
$1.0, u_\tau$


[0.03, 1]
p = 0
r ≤ 0
a
1.0, u1
[0.03, 1]
p = 1
¬(r ≤ 0)
0.97, u1
b
0.03, u2
0.03, u2
0.97, u1
[1, 1]
[0, 0]
p = 3
¬(r ≤ 0)
p = 1
r ≤ 0
aτ
1.0, uτ
c
1.0, u1
[0, 0]
p = 2
r ≤ 0
aτ
1.0, uτ


[0.03, 1]
p = 0
r ≤ 0
a
1.0, u1
[0.03, 1]
p = 1
¬(r ≤ 0)
0.97, u1
b
0.03, u2
0.03, u2
0.97, u1
[1, 1]
[0, 0]
p = 3
¬(r ≤ 0)
p = 1
r ≤ 0
aτ
1.0, uτ
c
1.0, u1
[0, 0]
p = 2
r ≤ 0
aτ
1.0, uτ


[0.03, 1]
$p = 0$
$r \leq 0$
a
$1.0, u_1$
[0.03, 1]
$p = 1$
$\neg(r \leq 0)$
b
$0.97, u_1$
$0.03, u_2$
$0.03, u_2$
$0.97, u_1$
[1, 1]
[0, 0]
$p = 3$
$\neg(r \leq 0)$
$p = 1$
$r \leq 0$
a_τ
$1.0, u_\tau$
c
$1.0, u_1$
[0, 0]
$p = 2$
$r \leq 0$
a_τ
$1.0, u_\tau$


$p = 0$
$r \leq 0$
$r \leq 1$
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$1.0, u_1$
$p = 1$
$\neg(r \leq 0)$
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$0.03, u_2$
$0.03, u_2$
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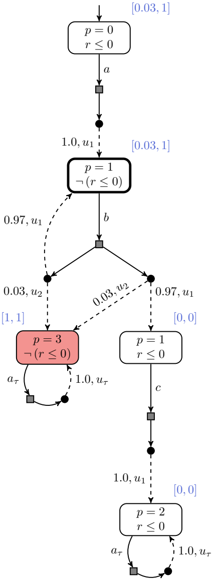
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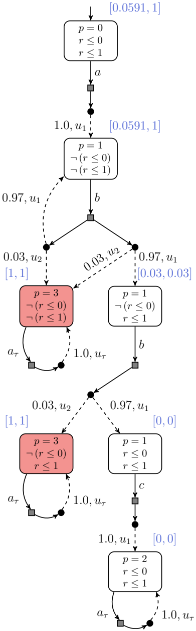
1.0, uτ


[0.0591, 1]
p = 0
r ≤ 0
r ≤ 1
a
1.0, u1
[0.0591, 1]
p = 1
¬(r ≤ 0)
¬(r ≤ 1)
0.97, u1
b
0.03, u2
0.03, u2
0.97, u1
[1, 1]
[0.03, 0.03]
p = 3
¬(r ≤ 0)
¬(r ≤ 1)
p = 1
¬(r ≤ 0)
r ≤ 1
aτ
1.0, uτ
b
0.03, u2
0.97, u1
[1, 1]
[0, 0]
p = 3
¬(r ≤ 0)
r ≤ 1
p = 1
r ≤ 0
r ≤ 1
aτ
1.0, uτ
c
1.0, u1
[0, 0]
p = 2
r ≤ 0
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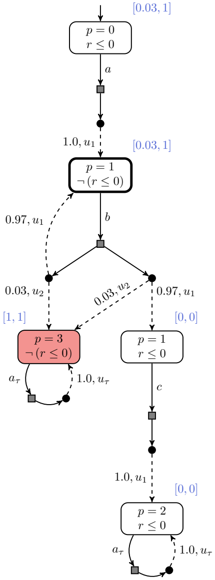
aτ
1.0, uτ



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p = 1
¬(r ≤ 0)
0.97, u1
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0.03, u2
0.03, u2
0.97, u1
[1, 1]
[0, 0]
p = 3
¬(r ≤ 0)
p = 1
r ≤ 0
aτ
1.0, uτ
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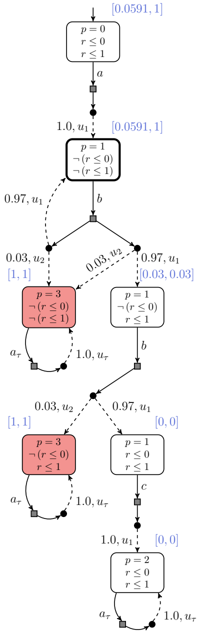

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0.03, u2
0.03, u2
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[1, 1]
[0.03, 0.03]
p = 3
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1.0, uτ
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r ≤ 1
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1.0, u1
[0, 0]
p = 2
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r ≤ 1
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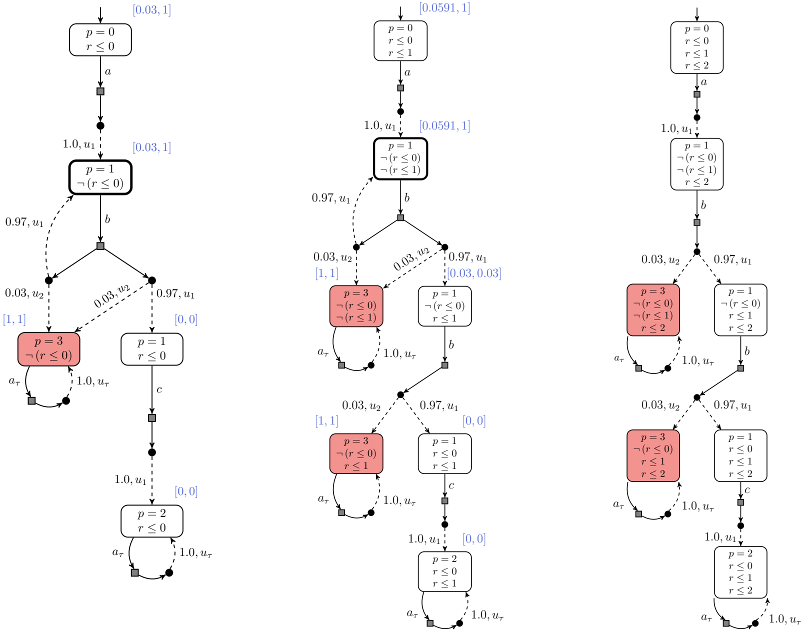
1.0, uτ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$p = 0$
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$p = 3$
$\neg(r \leq 0)$
$p = 1$
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a_τ
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$r \leq 0$
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$\neg(r \leq 1)$
$0.97, u_1$
b
$0.03, u_2$
$0.03, u_2$
$0.97, u_1$
[1, 1]
[0.03, 0.03]
$p = 3$
$\neg(r \leq 0)$
$\neg(r \leq 1)$
$p = 1$
$\neg(r \leq 0)$
$r \leq 1$
a_τ
$1.0, u_\tau$
b
$0.03, u_2$
$0.97, u_1$
[1, 1]
[0, 0]
$p = 3$
$\neg(r \leq 0)$
$r \leq 1$
$p = 1$
$r \leq 0$
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a_τ
$1.0, u_\tau$
c
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$r \leq 1$
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$r \leq 0$
$r \leq 1$
$r \leq 2$
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$\neg(r \leq 1)$
$r \leq 2$
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$0.03, u_2$
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$\neg(r \leq 0)$
$\neg(r \leq 1)$
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$r \leq 1$
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$1.0, u_\tau$
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$0.03, u_2$
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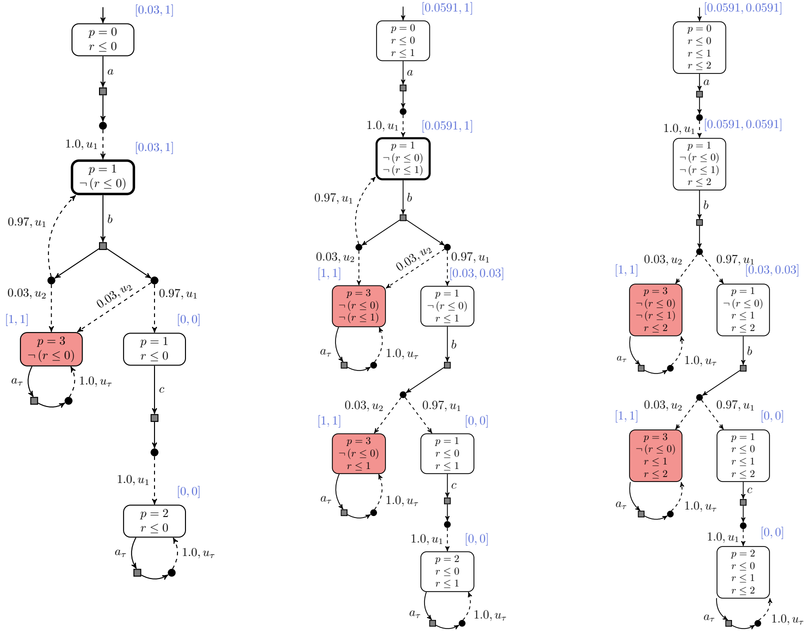
$p = 3$
$\neg(r \leq 0)$
$r \leq 1$
$r \leq 2$
$p = 1$
$r \leq 0$
$r \leq 1$
$r \leq 2$
a_τ
$1.0, u_\tau$
c
$1.0, u_1$
$p = 2$
$r \leq 0$
$r \leq 1$
$r \leq 2$
a_τ
$1.0, u_\tau$


[0.03, 1]
$p = 0$, $r \leq 0$
a
$1.0, u_1$
[0.03, 1]
$p = 1$, $\neg(r \leq 0)$
b
$0.97, u_1$
$0.03, u_2$
$0.03, u_2$
$0.97, u_1$
[1, 1]
[0, 0]
$p = 3$, $\neg(r \leq 0)$
$p = 1$, $r \leq 0$
a_τ
$1.0, u_\tau$
c
$1.0, u_1$
[0, 0]
$p = 2$, $r \leq 0$
a_τ
$1.0, u_\tau$
[0.0591, 1]
$p = 0$, $r \leq 0$, $r \leq 1$
a
$1.0, u_1$
[0.0591, 1]
$p = 1$, $\neg(r \leq 0)$, $\neg(r \leq 1)$
$0.97, u_1$
b
$0.03, u_2$
$0.03, u_2$
$0.97, u_1$
[1, 1]
[0.03, 0.03]
$p = 3$, $\neg(r \leq 0)$, $\neg(r \leq 1)$
$p = 1$, $\neg(r \leq 0)$, $r \leq 1$
a_τ
$1.0, u_\tau$
b
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[1, 1]
[0, 0]
$p = 3$, $\neg(r \leq 0)$, $r \leq 1$
$p = 1$, $r \leq 0$, $r \leq 1$
a_τ
$1.0, u_\tau$
c
$1.0, u_1$
[0, 0]
$p = 2$, $r \leq 0$, $r \leq 1$
a_τ
$1.0, u_\tau$
[0.0591, 0.0591]
$p = 0$, $r \leq 0$, $r \leq 1$, $r \leq 2$
a
$1.0, u_1$
[0.0591, 0.0591]
$p = 1$, $\neg(r \leq 0)$, $\neg(r \leq 1)$, $r \leq 2$
b
$0.03, u_2$
$0.97, u_1$
[1, 1]
[0.03, 0.03]
$p = 3$, $\neg(r \leq 0)$, $\neg(r \leq 1)$, $r \leq 2$
$p = 1$, $\neg(r \leq 0)$, $r \leq 1$, $r \leq 2$
a_τ
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$0.03, u_2$
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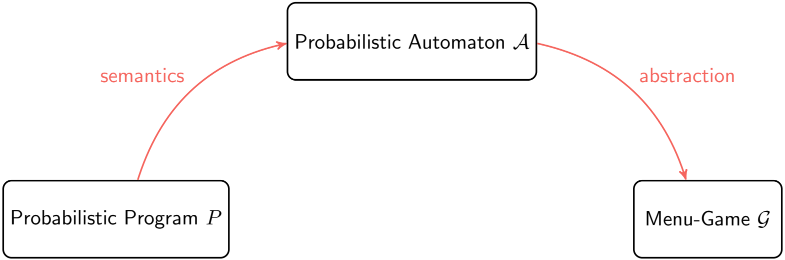
[1, 1]
[0, 0]
$p = 3$, $\neg(r \leq 0)$, $r \leq 1$, $r \leq 2$
$p = 1$, $r \leq 0$, $r \leq 1$, $r \leq 2$
a_τ
$1.0, u_\tau$
c
$1.0, u_1$
[0, 0]
$p = 2$, $r \leq 0$, $r \leq 1$, $r \leq 2$
a_τ
$1.0, u_\tau$

Motivating SMT-based Construction



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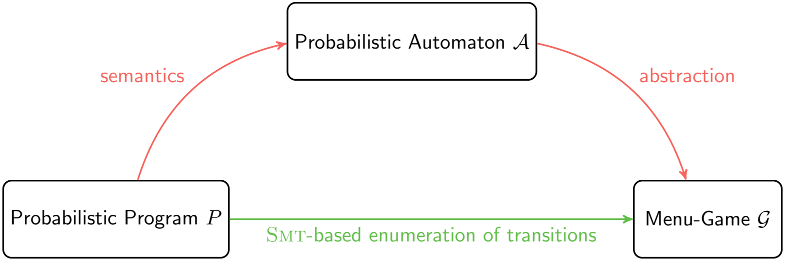

SMT-based Construction (Example)

Consider

$$[a]\ x > 0 \rightarrow 0.7 : (x' = x + 1) + 0.3 : (y' = x)$$

$$\mathcal{P} = \{x \text{ is odd}, y \text{ is odd}\}$$

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Abstract Transition Constraint

$x > 0$

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$$\wedge\ (b_0^{src} \Leftrightarrow x \text{ is odd}) \wedge (b_1^{src} \Leftrightarrow y \text{ is odd})$$

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Interpretation

- $(b_0^{src}, b_1^{src}) = (1, 0)$
- $(b_0^{dst_1}, b_1^{dst_1}) = (0, 0)$
- $(b_0^{dst_2}, b_1^{dst_2}) = (1, 1)$

SMT-based Construction (Example)

Consider

$$[a]\ x > 0 \rightarrow 0.7 : (x' = x + 1) + 0.3 : (y' = x)$$

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$$\wedge\ (b_0^{src} \Leftrightarrow x \text{ is odd}) \wedge (b_1^{src} \Leftrightarrow y \text{ is odd})$$
$$\wedge\ (b_0^{dst_1} \Leftrightarrow x + 1 \text{ is odd}) \wedge (b_1^{dst_1} \Leftrightarrow y \text{ is odd})$$
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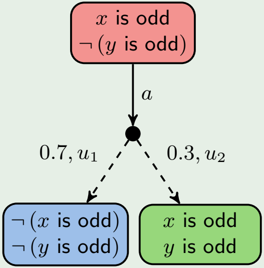

Relevant Predicates

Observation

Commands are often only related to a subset of all predicates

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$\Rightarrow$ irrelevant destination predicates retain their value

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Simplify Transition Constraint

$$[a]\ x > 0 \rightarrow 0.7 : (x' = x + 1) + 0.3 : (y' = x)$$

$$\begin{aligned} x > 0 &\wedge (b_0^{src} \Leftrightarrow x \text{ is odd}) \wedge (b_1^{src} \Leftrightarrow y \text{ is odd}) \\ &\wedge (b_0^{dst_1} \Leftrightarrow x+1 \text{ is odd}) \wedge (b_1^{dst_1} \Leftrightarrow y \text{ is odd}) \\ &\wedge (b_0^{dst_2} \Leftrightarrow x \text{ is odd}) \wedge (b_1^{dst_2} \Leftrightarrow x \text{ is odd}) \end{aligned}$$

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$x > 0 \wedge (b_0^{src} \Leftrightarrow x$ is odd$) \wedge$ ~~$(b_1^{src} \Leftrightarrow y$ is odd$)$~~

$\wedge\ (b_0^{dst_1} \Leftrightarrow x + 1$ is odd$) \wedge$ ~~$(b_1^{dst_1} \Leftrightarrow y$ is odd$)$~~

$\wedge$ ~~$(b_0^{dst_2} \Leftrightarrow x$ is odd$)$~~ $\wedge\ (b_1^{dst_2} \Leftrightarrow x$ is odd$)$

$\Rightarrow$ extend solutions with $b_1^{src} \Leftrightarrow b_1^{dst_1}$ and $b_0^{src} \Leftrightarrow b_0^{dst_2}$

Reachable Blocks as Constraint

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Refinement can only split blocks but never introduce “new” ones

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New Solutions are Extensions

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Idea

Extend transition constraint with reachable blocks constraints

Other optimisations

Variables' Ranges $\Rightarrow$ extend constraint with variables' domains

Exploit Incrementality

Predicate Decomposition

Unrelated Commands

Other optimisations

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Observations

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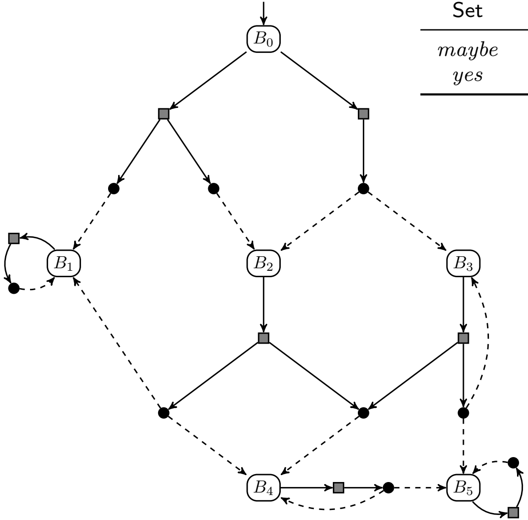

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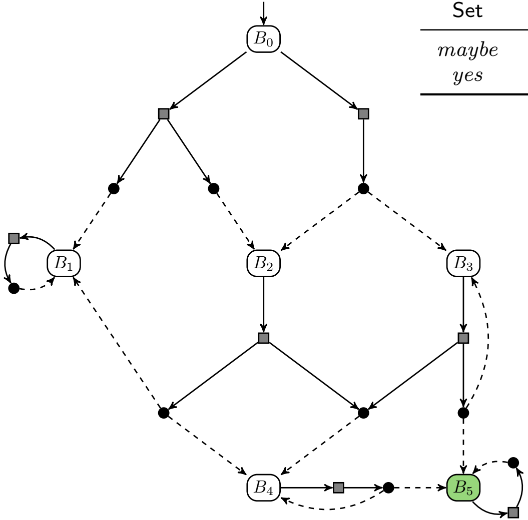

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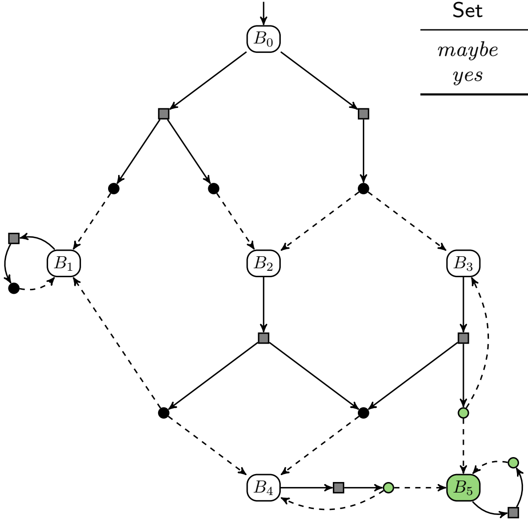

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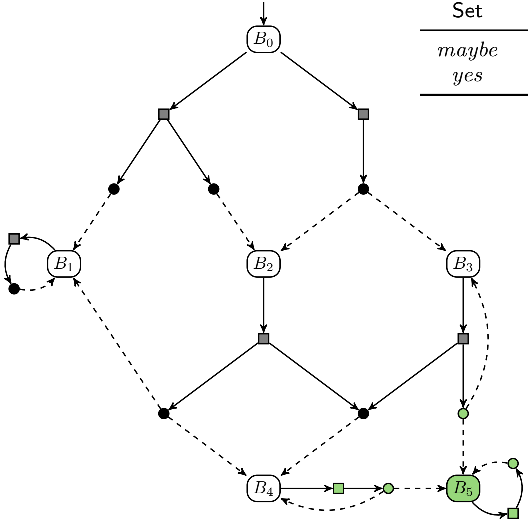

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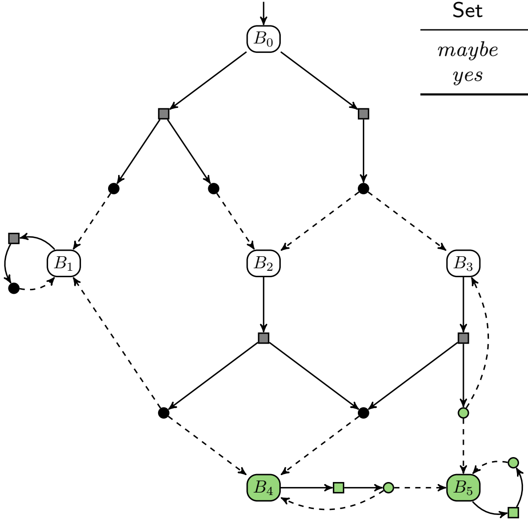

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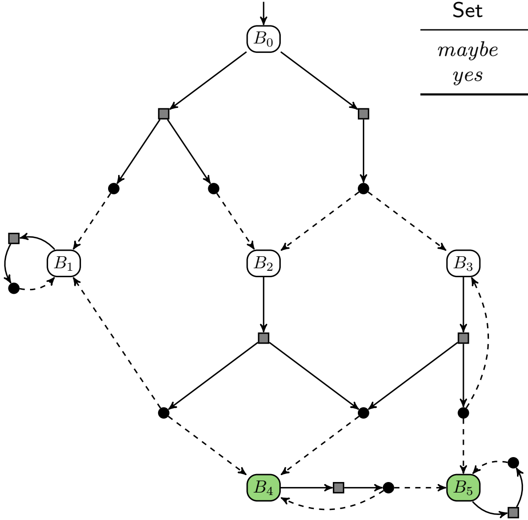

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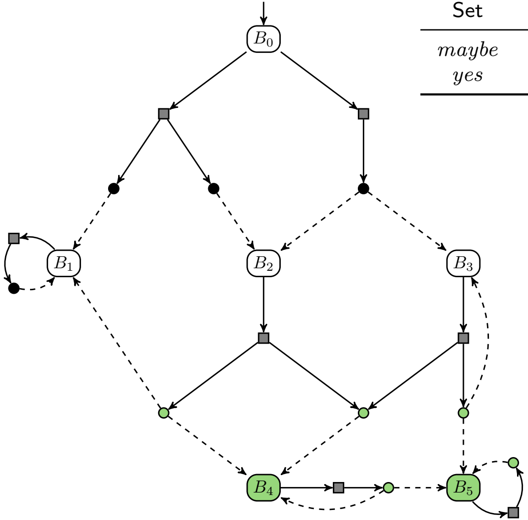

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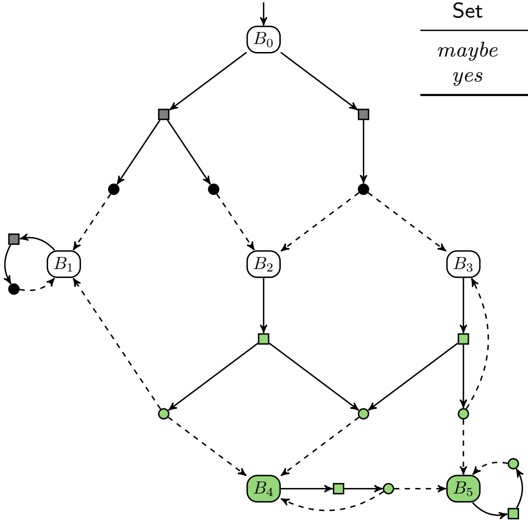

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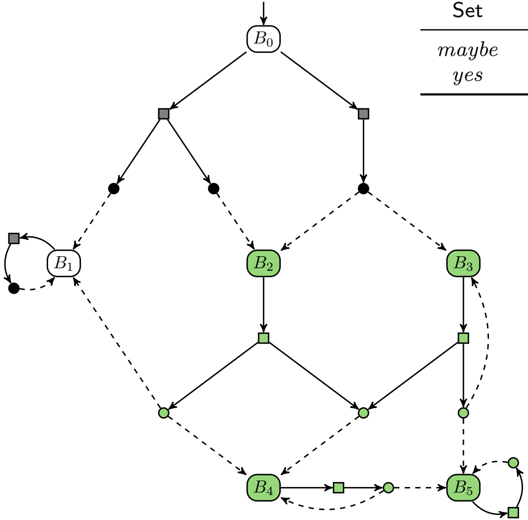

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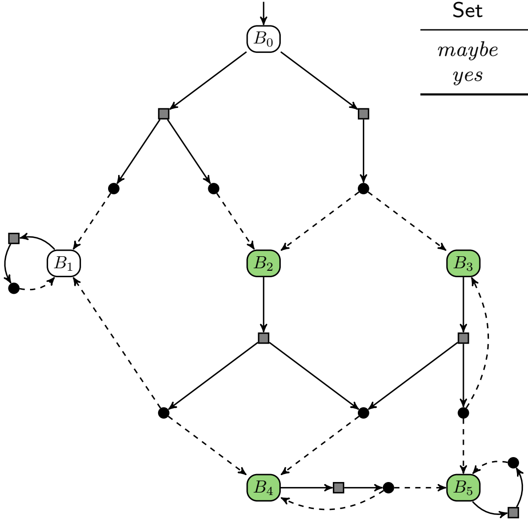

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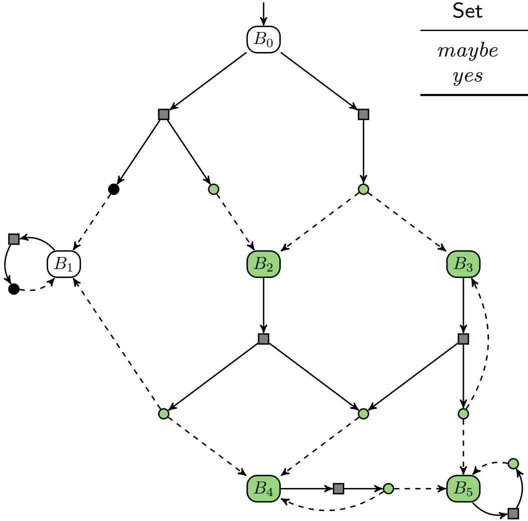

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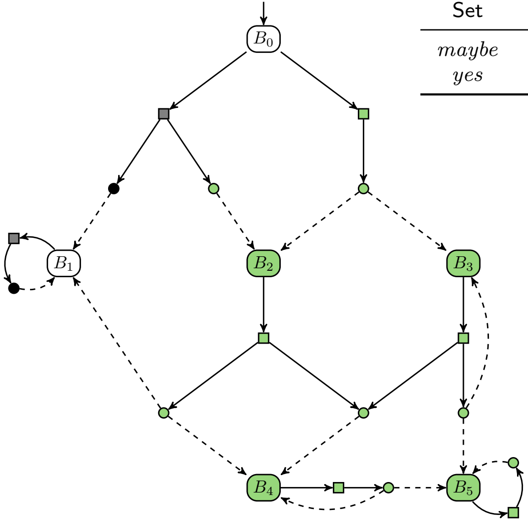

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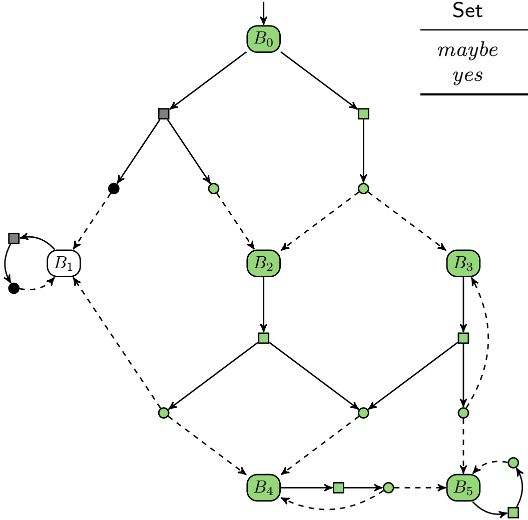

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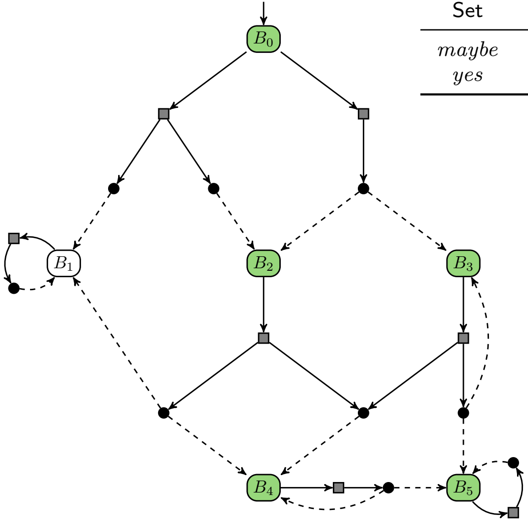

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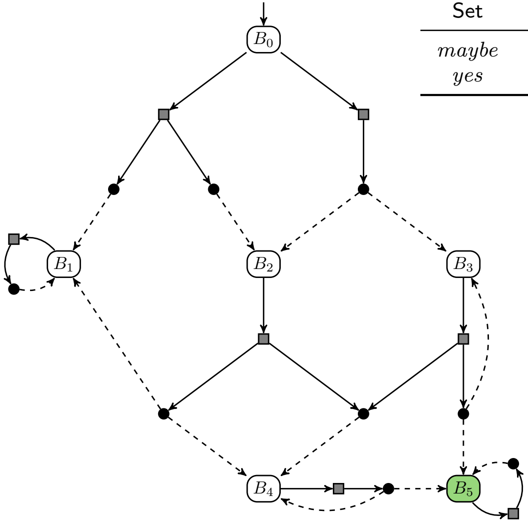

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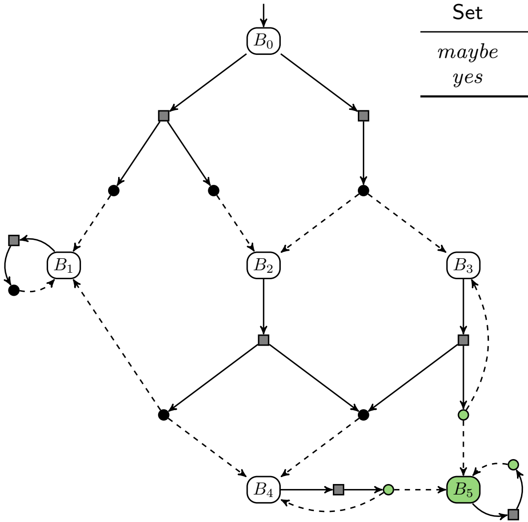

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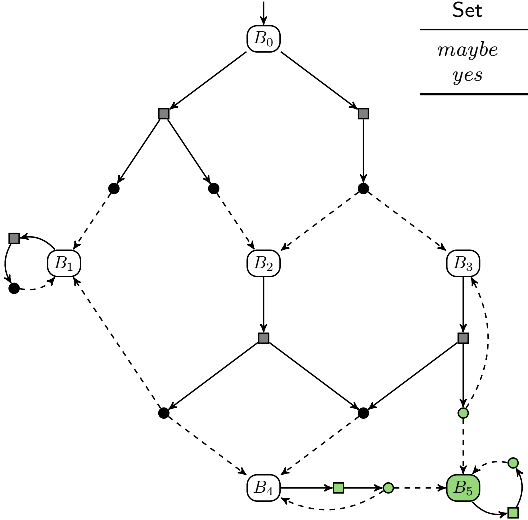

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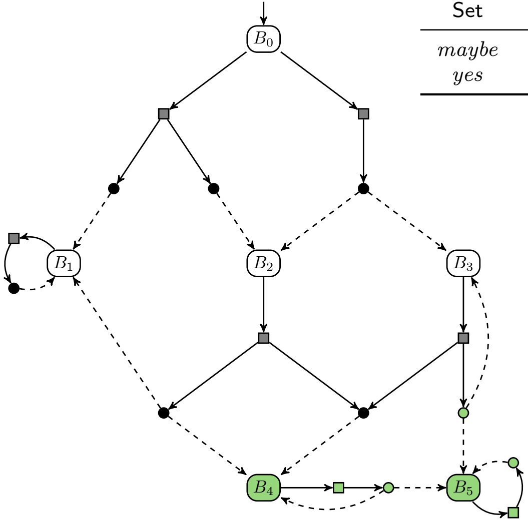

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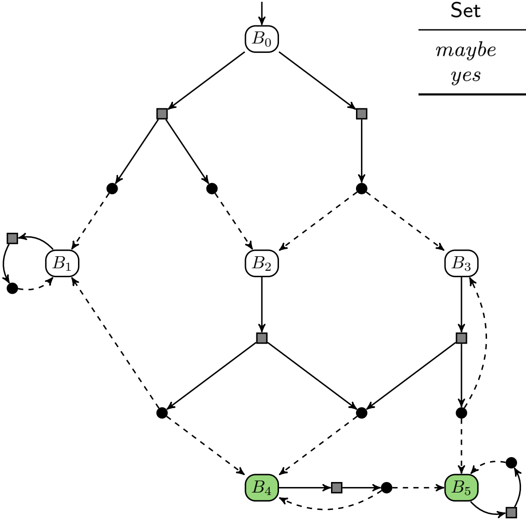

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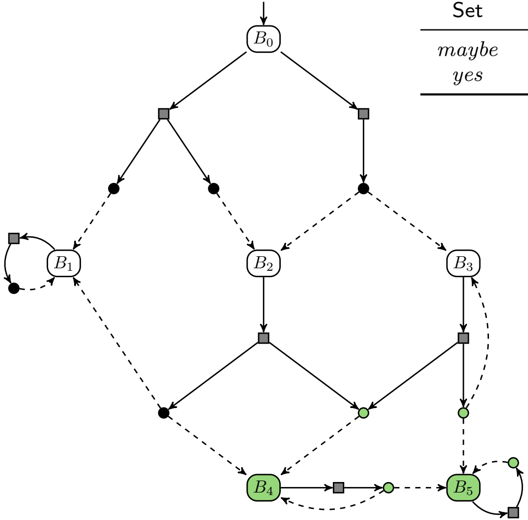

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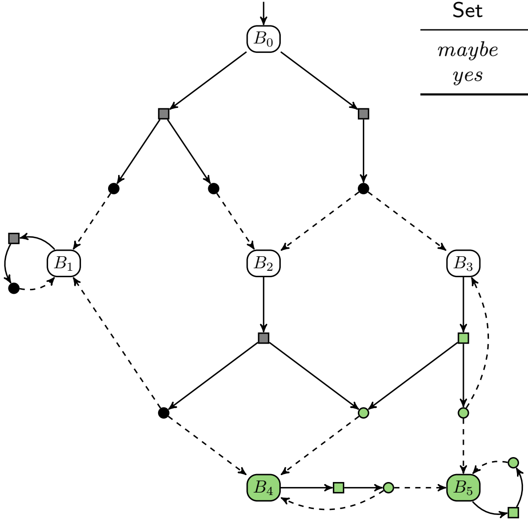

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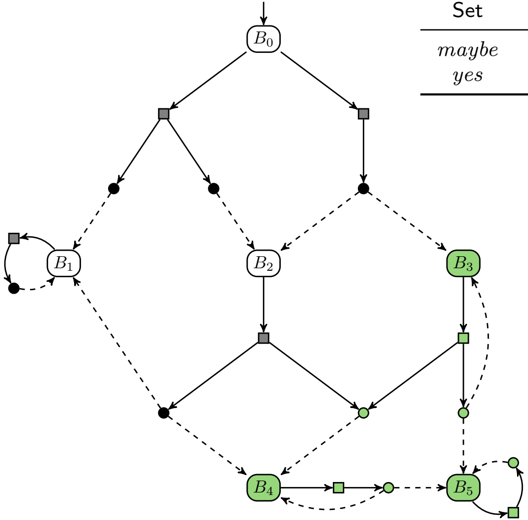

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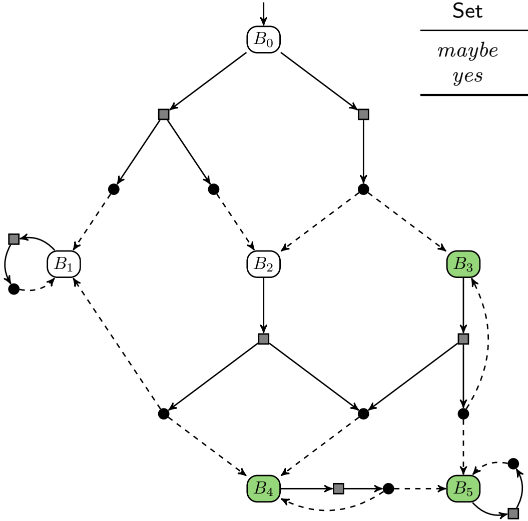

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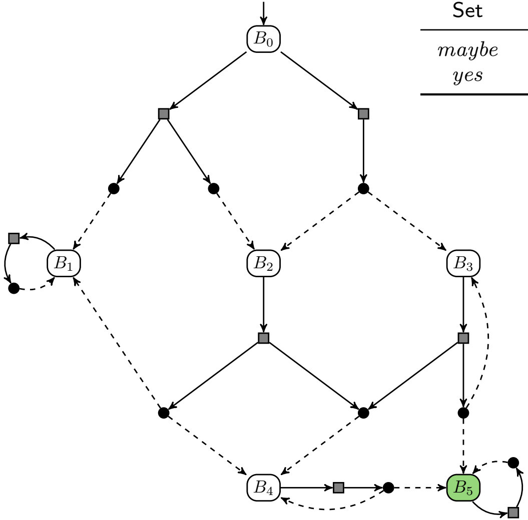

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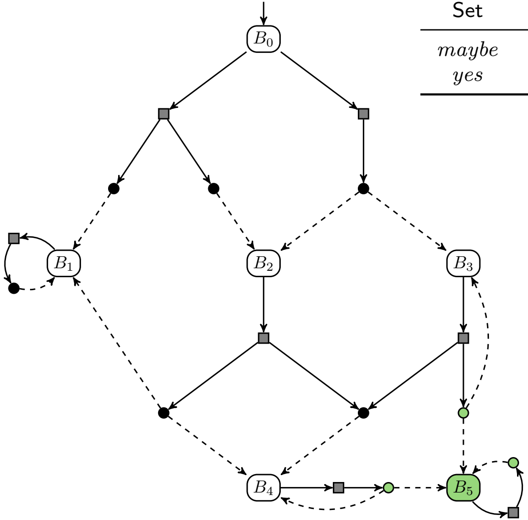

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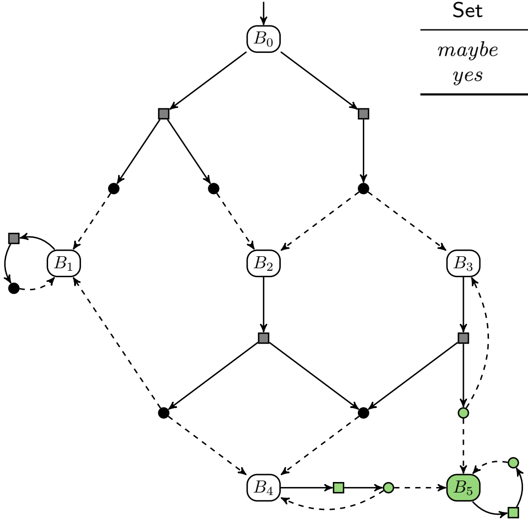

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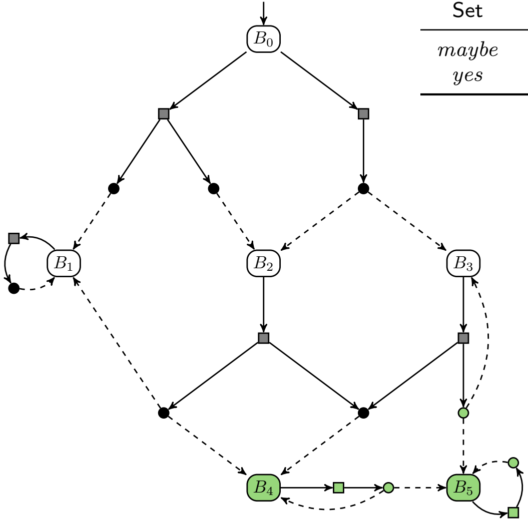

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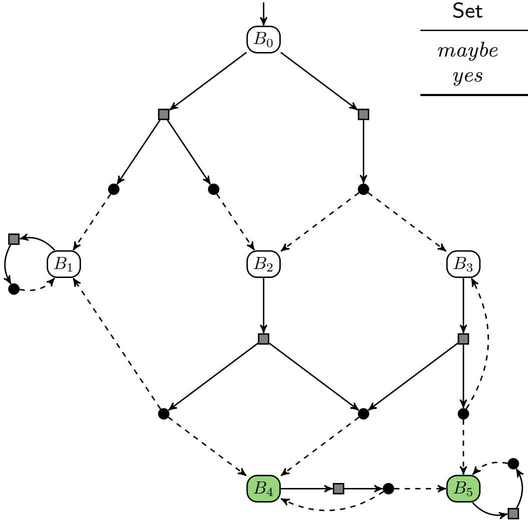

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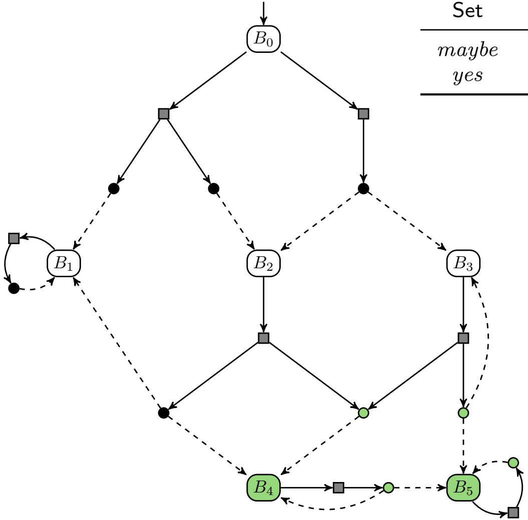

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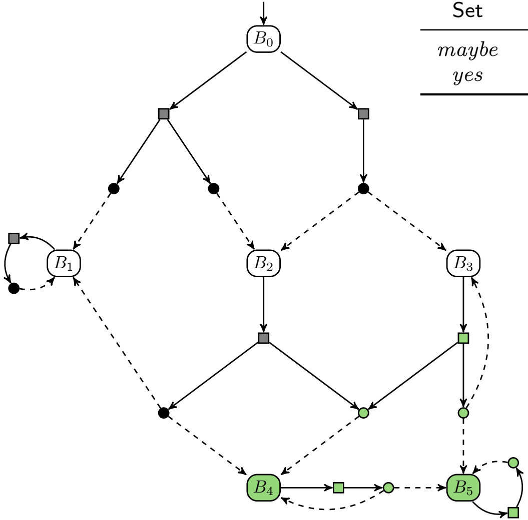

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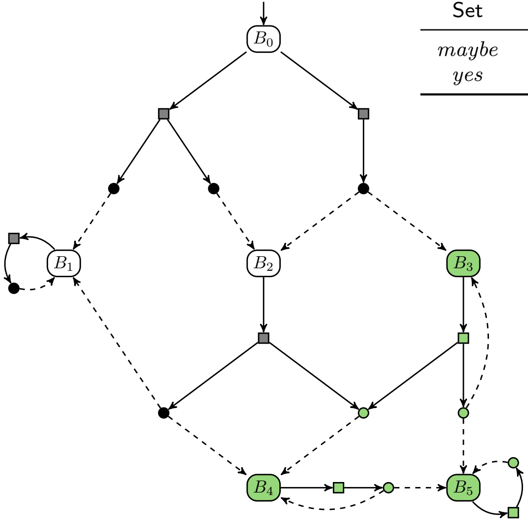

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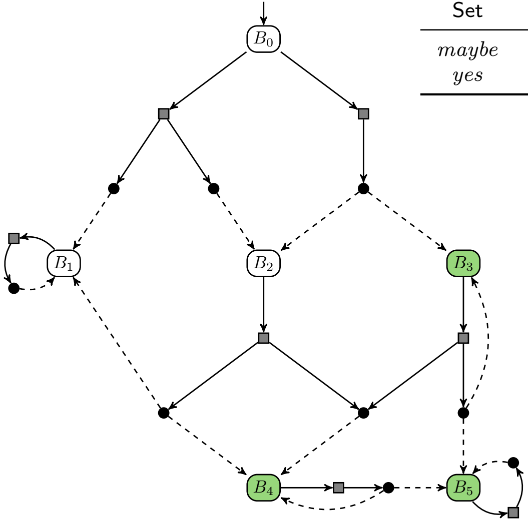

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- avoid starting value iteration from scratch

Remove goal transitions

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⇒ reuse previous refinement iteration results (where applicable)

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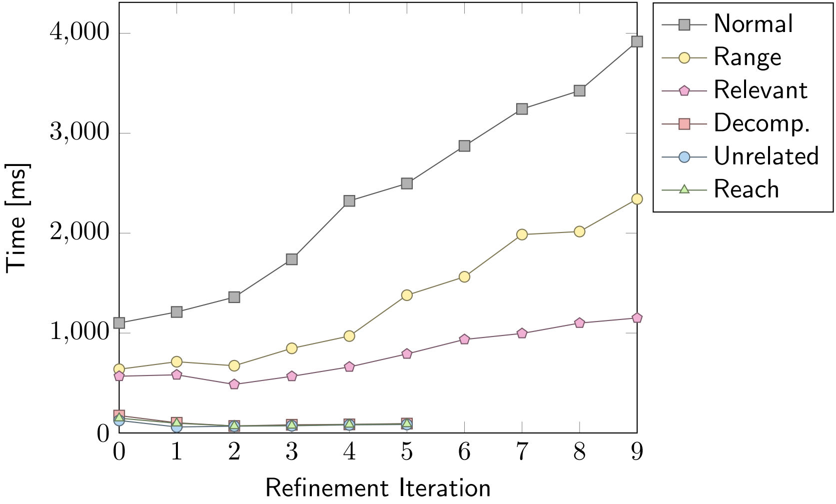
- evaluated symbolic vs. explicit analysis (memory usage & run time)

Consensus (Abstraction)



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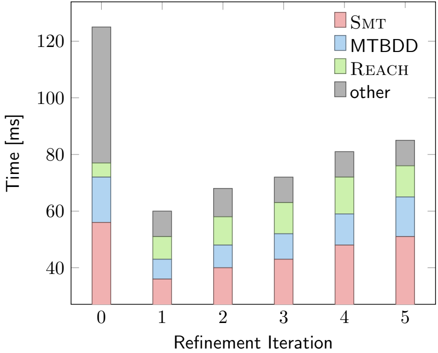
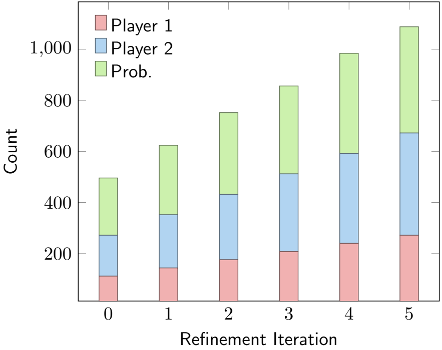
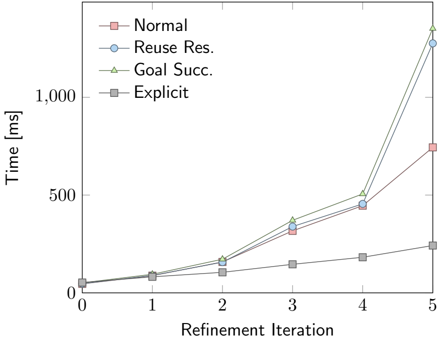
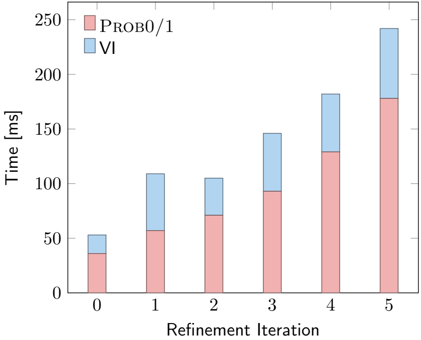


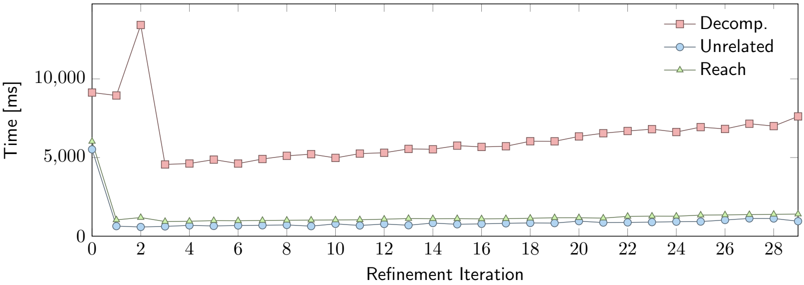


Consensus (Analysis)





WLAN (Abstraction)



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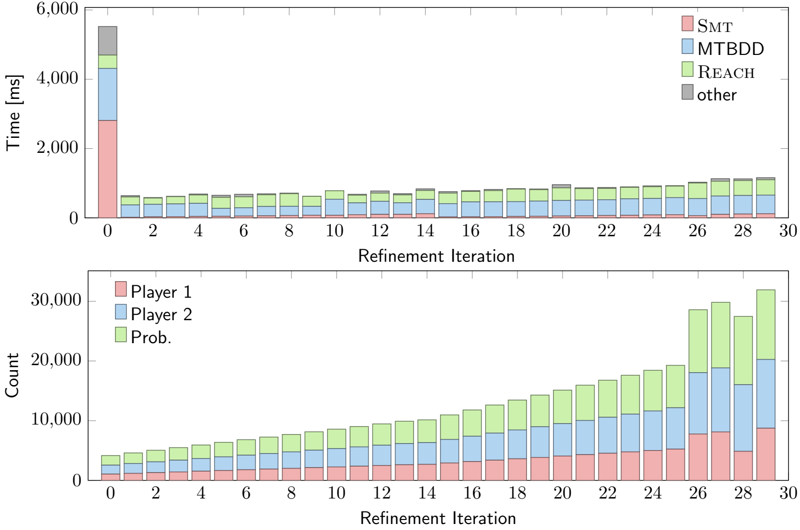
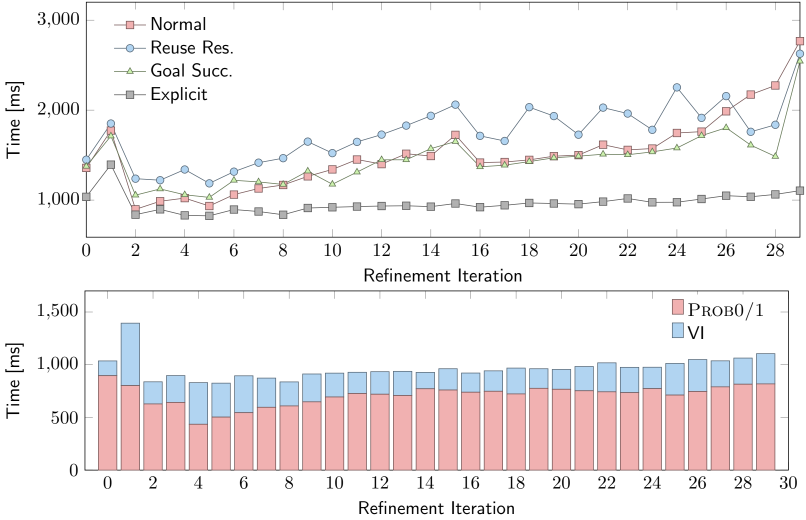
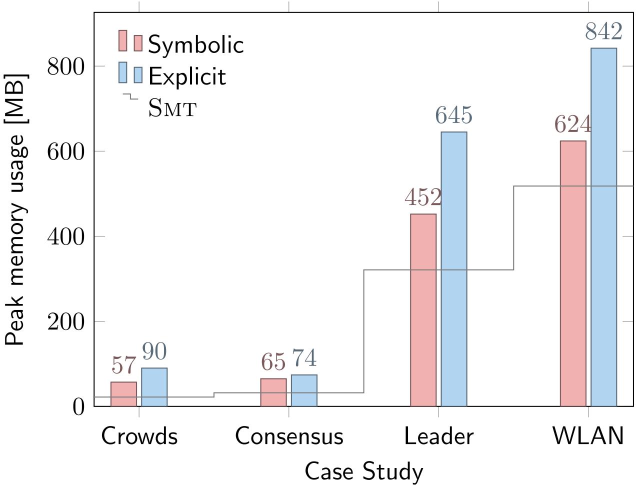


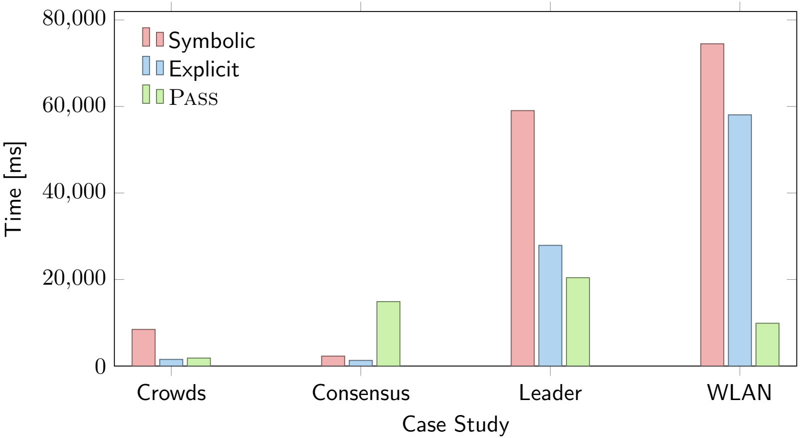
WLAN (Analysis)



Memory usage



Symbolic vs. Explicit vs. PASS



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- exploit modularity of probabilistic programs

Thanks for your attention!

Interested in details? Suggestions?